DETAILED

DERIVATION

JENSEN MODEL

ED- JULY 2012.

Formules Jensen model with SF

$$k = \frac{1}{1 + 1{,}4\,\frac{SF}{c}} \qquad p = \frac{\Delta w}{V_c + V_w + V_{SF}}$$

$$V_{cs}\,(\text{power}) = \rho_c \cdot \Delta V_{Rc}\,(1-p) \cdot \alpha \qquad \Delta V_R = \frac{6{,}4\,ml}{100\,g\ c\ \text{[illegible]}}$$

$$p = \frac{V_w}{V_w + V_c} \qquad 1 - p = \frac{V_c}{V_w + V_c}$$

$$V_{cs}\,(\text{jensen}) = k \cdot \left(0{,}2 + 0{,}7 \cdot \frac{SF}{c}\right)(1-p) \cdot \alpha$$

$$p = \frac{V_w}{V_w + V_c + V_{SF}}$$

$$= \frac{m_c}{m_w} \times \frac{m_w}{m_c} \times \frac{V_w}{V_w + V_c + V_{SF}}$$

$$= \frac{w}{c} \times \frac{1}{1 + \frac{V_c}{V_w} + \frac{V_{SF}}{V_w}} \cdot \frac{m_c}{m_w}$$

$$= \frac{w}{c} \times \frac{1}{\frac{w}{c} + \frac{m_w}{m_c} \cdot \frac{V_c}{V_w} + \frac{V_{SF}}{V_w} \cdot \frac{m_w}{m_c}}$$

$$= \frac{w}{c} \times \frac{1}{\frac{w}{c} + \frac{\rho_w}{\rho_c} + \underbrace{\frac{V_{SF}}{m_c} \cdot \rho_w}_{\frac{V_{SF}}{m_{SF}} \times \frac{m_{SF}}{m_c} \times \rho_w}}$$

$$= \frac{\frac{w}{c}}{\frac{w}{c} + \frac{\rho_w}{\rho_c} + \frac{SF}{c} \times \frac{\rho_w}{\rho_{SF}}} \quad \text{ok!}$$

$$V_{cs}\,(\text{POWERS}) = \frac{m_c}{V_c} \cdot \Delta V_{RC}\,(1-p) \cdot \alpha$$

$$= \frac{m_c}{\cancel{V_C}} \cdot \Delta V_{RC} \cdot \frac{\cancel{V_C}}{V_W + V_C} \cdot \alpha$$

$$= m_c \cdot \alpha \cdot \frac{\Delta V_{RC}}{V_W + V_C}$$

$$\Delta V_{RC} = \frac{ml}{100g\ \text{c hydrate}} \qquad \text{OK}$$

$$V_{cs}\,(\text{jensen}) = k\left(0{,}2 + 0{,}7\,\frac{SF}{c}\right)(1-p) \cdot \alpha$$

$$1 - p = \frac{V_C + V_{SF}}{V_C + V_W + V_{SF}}$$

$$V_{cs}\,(\text{jensen}) = \frac{1}{1 + 1{,}4\,\frac{SF}{c}}\left(0{,}2 + 0{,}7\,\frac{SF}{c}\right)(1-p) \cdot \alpha$$

$$V_{cs}\,(\text{jensen}) = V_{cs}(c) + V_{cs}(SF)$$

$$0{,}2 \Rightarrow \rho_C \times 6{,}4 \times 10^{-5} = \underline{\underline{0{,}2}}$$

$$\rho_{SF} \times 22 \times 10^{-5} = \underline{\underline{0{,}48}}$$

$$V_{cs}\,(\text{jensen}) = \frac{1}{1 + 1{,}4 \cdot \frac{m_{SF}}{m_C}}\left(0{,}2 + 0{,}7\,\frac{m_{SF}}{m_C}\right)(1-p).$$

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$$V_{cs}(c+SF) = m_c \cdot \boxed{\alpha^c} \cdot \frac{\Delta V_{Rc}}{V_w + V_c + V_{SF}} + m_{SF} \cdot \boxed{\alpha^c} \cdot \frac{\Delta V_{SF}}{V_w + V_c + V_{SF}}$$

proportionality!

see Waller!

$$1 - P = \frac{V_c + V_{SF}}{V_w + V_c + V_{SF}}$$

$$\Rightarrow V_{cs}(c+SF) = m_c \cdot \alpha \cdot \frac{\Delta V_{Rc}}{V_c + V_{SF}} \cdot 1-P + m_{SF} \cdot \alpha \cdot \frac{\Delta V_{SF}}{V_c + V_{SF}} (1-P)$$

$$= \alpha(1-P) \cdot \left(\frac{\Delta V_{Rc}}{V_c + V_{SF}} m_c + \frac{\Delta V_{SF} \cdot m_{SF}}{V_c + V_{SF}} \right)$$

$$= \frac{\alpha(1-P)}{V_c + V_{SF}} (m_c \cdot \Delta V_{Rc} + m_{SF} \Delta V_{SF})$$

$$= \frac{m_c}{V_c + V_{SF}} (1-P) \cdot \alpha \left(\Delta V_{Rc} + \Delta V_{SF} \cdot \frac{SF}{c} \right).$$

$$= \frac{\rho_c}{1 + \frac{V_{SF}}{V_c}} (1-P) \cdot \alpha \left(\Delta V_{Rc} + \Delta V_{SF} \cdot \frac{SF}{c} \right)$$

$$V_{CS}(C+SF) = \frac{1}{1+\frac{V_{SF}}{m_{SF}}\cdot\frac{m_{SF}}{m_C}\cdot\frac{m_C}{V_C}}\cdot(1-p)\cdot\alpha\,.$$

$$\times\ \rho_C\left(\Delta V_{RC} + \Delta V_{SF}\cdot\frac{SF}{C}\right).$$

$$= \left(\frac{1}{1+\frac{\rho_C}{\rho_{SF}}\cdot\frac{SF}{C}}\right)\cdot(1-p)\cdot\alpha\,.$$

$$\times\ \underbrace{\rho_C\left(\Delta V_{RC} + \Delta V_{SF}\,\frac{SF}{C}\right)}$$

$$\left(\underbrace{\rho_C\cdot\Delta V_{RC}}_{0,2} + \underbrace{\rho_C\cdot\Delta V_{SF}}_{350\times 22\times 10^{-5}\,=\,0,69}\cdot\frac{SF}{C}\right)$$

ok!!

$$V_{GW} = \frac{(V_{GWC} + V_{GWSF})}{V_C + V_W + V_{SF}}$$

$$= \frac{\alpha\left(\frac{m_C \cdot \Delta m_{G,W,C}}{\rho_W} + \frac{m_{SF}}{\rho_W} \cdot \Delta m_{G,W,SF}\right)}{V_C + V_W + V_{SF}}$$

$$= \frac{m_C \cdot (1-p) \cdot \alpha}{V_C + V_{SF}} \cdot \left(\frac{\Delta m_{GWC}}{\rho_W} + \frac{\Delta m_{GWSF}}{\rho_W} \cdot \frac{m_{SF}}{m_C}\right)$$

$$= \frac{(1-p) \cdot \alpha}{1 + \frac{V_{SF}}{V_C}} \cdot \left(\Delta m_{GWC} \cdot \frac{\rho_C}{\rho_W} + \Delta m_{GWSF} \cdot \frac{m_{SF}}{m_C} \cdot \frac{\rho_C}{\rho_W}\right.$$

$$= (1-p) \cdot \alpha \cdot k \left(\frac{\rho_C}{\rho_W} \cdot \Delta m_{GWC} + \frac{m_{SF}}{m_C} \cdot \frac{\rho_C}{\rho_W} \cdot \Delta m_{GWSF}\right.$$

$$\frac{3150}{1000} \times 0{,}19 = 0{,}6$$

ok!

$$\frac{3150}{1000} \times 0{,}50 = 1{,}60$$

$$V_{cw} = \frac{(V^*_w - V^*_{Gw} - V^*_{CHBw})}{V_c + V_{SF} + V_w}.$$

$$= p - V_{Gw} - \frac{V^*_{CHBW}}{(V_w + V_c + V_{SF})}$$

$$V^*_{CHBW} = V^*_{CHBWc} + \underbrace{V^*_{CH\cancel{BW}SF}}_{\to 0} \text{ (chào}$$

$$= \frac{\Delta m_{CHBWc}[g/g] \cdot m_c \cdot \alpha}{\rho_w}$$

$$V_{cw} = p - V_{Gw} - \frac{\Delta m_{CHBWc} \cdot m_c \cdot \alpha (1-p)}{(V_c + V_{SF}) \cdot \rho_w}$$

$$= p - V_{Gw} - \frac{\Delta m_{CHBWc} \cdot \alpha (1-p)}{\rho_w} \cdot \frac{m_c}{V_c + V_{SF}}$$

$$= p - V_{Gw} - \frac{\Delta m_{CHBWc} \cdot \alpha \cdot (1-p)}{\rho_w} \cdot \underbrace{\boxed{\frac{\rho_c \cdot 1}{1 + \frac{\rho_c}{\rho_{SF}} \cdot \frac{SF}{C}}}}_{K.}$$

$$= p - (1-p) \cdot \alpha \cdot K \Big(\underbrace{\frac{\rho_c}{\rho_w} \cdot \Delta m_w G_{wc}}_{①} + \underbrace{\frac{\rho_c}{\rho_w} \cdot \Delta m_{GSF} \cdot \frac{SF}{C}}_{②} + \underbrace{\frac{\rho_c}{\rho_w} \Delta m_{CHBW}}_{①} \Big)$$

$$① = \frac{\rho_c}{\rho_w} [\Delta m_w G_{wc} + \Delta m_{CHBWc}]$$

$$= \frac{3150}{1000} [0{,}19 + \underline{\underline{0{,}254}}] = 1{,}399$$

(1,32 avec 0,23 g/g C.)

$$② = \frac{3150}{1000} \cdot 0{,}50 = 1{,}5\underline{\underline{7 \text{ OK}}}!$$

V_{GS} = POWERS

$$= \left(1 - \rho_c \cdot \Delta V_{RCC} + \frac{\rho_c}{\rho_w} \cdot 0{,}23\right)(1-p) \cdot \alpha$$

$$V_{CHBW} = \frac{\Delta m_{CHBWC}}{\rho_w} \cdot \frac{m_c \cdot \alpha}{V_w + V_c + V_{SF}}$$

$$V_{GS} = \frac{V^{*\alpha}_{GS,c} + V_{GS}{}^{*}{}_{SF} + V^{*\alpha}_{CHBW} - V^{*\alpha}_{cs}}{V_c + V_{SF} + V_w}$$

$$= \frac{\alpha \frac{m_c}{\rho_c} + \alpha \frac{m_{SF}}{\rho_{SF}} + \alpha \frac{\Delta m_{CHBWC}}{\rho_w} \cdot m_c}{V_c + V_{SF} + V_w} - V_c$$

$$= \frac{\alpha(1-p)\left[\frac{m_c}{\rho_c} + \frac{m_{SF}}{\rho_{SF}} + \frac{\Delta m_{CHBWC}}{\rho_w} \cdot m_c\right]}{V_{SF} + V_c} - V_{cs}$$

$$= \frac{\alpha(1-p) \cdot m_c}{V_{SF} + V_c}\left(\frac{\cancel{m_c}\,1}{\rho_c} + \frac{m_{SF}}{\cancel{m_c}} \cdot \frac{1}{\rho_{SF}} + \frac{\Delta m_{CHW}}{\rho_w} \ldots\right.$$

$$- V_{cs}$$

$$= \alpha(1-p) \cdot \rho_c \cdot K \cdot \left(\frac{1}{\rho_c} + \frac{m_{SF}}{\cancel{m_c}} \cdot \frac{1}{\rho_{SF}} + \frac{\Delta m_{C}\ldots}{\rho_w} \ldots\right.$$

$$- V_{cs}$$

$$= (1-p) \cdot \alpha \cdot K \left(1 + \frac{\rho_c}{\rho_{SF}} \frac{m_{SF}}{m_c} + \Delta m_{CHBW} \cdot \frac{\rho_c}{\rho_w}\right)$$

$$- (1-p) \cdot \alpha \cdot K \left(\rho_c \cdot \Delta V_{RCC} + \rho_c \cdot \Delta V_{RCSF} \frac{m_{SF}}{m_c}\right)$$

$\Rightarrow V_{GS} =$

$\hookrightarrow (1-p).\alpha.k \left[\overset{ok}{1} + \overset{ok}{\Delta m_{CHBW}} \cdot \frac{\rho_C}{\rho_W} - \rho_C \cdot \overset{ok}{\Delta V_{RCC}} \right.$

$\left. + \frac{SF}{C} \left[\frac{\rho_C}{\rho_{SF}} - \rho_C \cdot \Delta V_{RCSF} \right] \right]$

(A) (B)

$(A) = \underbrace{1 + 0{,}254 \cdot \frac{3150}{1000}}_{1{,}8} - \underbrace{3150 \cdot 6{,}4 \cdot 10^{-5}}_{0{,}2016}$

$= 1{,}599$! ok (1,6)

$(B) = \frac{3150}{2290} - 3150 \times 20 \times 10^{-5}$

20 mL, $\rho_{SF} = 2200$.

$= 0{,}80$? manque qqme chose !

$= 0{,}707$ ok avec $\boxed{SF\ \Delta V = 22\ mL/100g \quad \rho_{SF} = 2250\ kg/m^3}$

$\frac{\rho_C}{\rho_{SF}} \times \frac{m_{SF}}{m_C} = \frac{\cancel{m_C}}{V_C} \times \frac{V_{SF}}{\cancel{m_{SF}}} \cdot \frac{\cancel{m_{SF}}}{\cancel{m_C}}$

$= \frac{V_{SF}}{V_C}$

V_c = non hydrated

$$= \frac{(1-\alpha).m_c/\rho_c}{V_w+V_c+V_{SF}}$$

$$= (1-\alpha).\frac{m_c}{\rho_c}.\frac{(1-p)}{V_c+V_{SF}} = \left((1-\alpha).\frac{m_c}{\rho_c}.\frac{(1-p)}{V_c+V_{SF}}\right)$$

$$= (1-\alpha).(1-p).\frac{m_c/\rho_c}{V_c+V_{SF}}$$

$$= (1-\alpha).(1-p).\frac{\rho_c/\rho_c}{1+\frac{V_{SF}}{V_c}}$$

$$= \underline{\underline{(1-\alpha)(1-p)}}.k. \quad \text{ok!}$$

V_{SF} = non hydrated

$$= (1-\alpha).\frac{m_{SF}}{\rho_{SF}}.\frac{1}{V_w+V_c+V_{SF}}$$

$$= (1-\alpha).\frac{m_{SF}}{\rho_{SF}} \times \frac{(1-p)}{V_c+V_{SF}}$$

$$= (1-\alpha).\frac{SF}{C}.\frac{(1-p)}{\rho_{SF}}.\frac{m_c}{V_c+V_{SF}}$$

$$= \underline{\underline{(1-\alpha)(1-p).\frac{SF}{C}.\frac{\rho_c}{\rho_{SF}}.k}} \approx 1{,}\underline{\underline{43.}} \text{ ok!}$$

Calculation of $\alpha\, max$

Closed system: $V_{GW} = 0$

$$\alpha\, max \Rightarrow P - (1-P)\,\alpha\, K\left(\frac{\rho c}{\rho w}\cdot \Delta m_{WGWC} + \frac{\rho c}{\rho w}\cdot \Delta m_{CHBW\ldots} + \frac{SF}{C}\cdot\frac{\rho c}{\rho w}\,\Delta m_{GWSF}\right) = \ldots$$

$$\Rightarrow \alpha\, max, ~~closed~~ = \frac{P}{(1-P)\cdot K\left(\frac{\rho c}{\rho w}\,\Delta m_{WGWC} + \frac{\rho c}{\rho w}\cdot \Delta m_{CHBWC} + \frac{SF}{C}\cdot\frac{\rho c}{\rho w}\cdot\Delta m_{GWSF}\right)}.$$

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open system:

$\alpha = \alpha\, max$ for $V_{GW} + V_{GS} + V_{UC} + V_{USF} = 1$
(Jensen Book p 163)

~~α $V_{GW} = [V_{GS}]$~~

α max, open:

$$V_{GW} + V_{GS} + V_{UC} + V_{USF} = 1.$$

$$(1-p).\alpha.K.\left[\frac{\rho c}{\rho w}.\Delta m_{GWC} + \left(\frac{m_{SF}}{m_C}\right).\frac{\rho c}{\rho w}.\Delta m_{GWSF} \right]$$

$$+ (1-p).\alpha.K \left[1 + \Delta m_{CHDWC}.\frac{\rho c}{\rho w} - \rho c.\Delta V_{RCC} + \left(\frac{m_{SF}}{m_C}\right).\left(\frac{\rho c}{\rho_{SF}} - \rho c.\Delta V_{RCSF} \right) \right]$$

$$+ (1-p)(1-\alpha).K + (1-p)(1-\alpha).\frac{m_{SF}}{m_C}.\frac{\rho c}{\rho_{SF}}.K$$

$$\left(K = \frac{1}{1 + \frac{\rho c}{\rho_{SF}}.\frac{m_{SF}}{m_C}} \right)$$

$$\Rightarrow (1-p).\alpha.K \left[A + B.\frac{m_{SF}}{m_C} \right]$$

$$+ (1-p)(1-\alpha)K + (1-p).(1-\alpha).K.\frac{m_{SF}}{m_C}.\frac{\rho c}{\rho_{SF}} = 1$$

$$\Rightarrow (1-p).\cancel{\alpha.K}\left(A + B\frac{m_{SF}}{m_C} \right)$$

$$+ \cancel{(1-p).K} - \cancel{(1-p).\alpha.K}$$

$$+ \cancel{(1-p).K}.\frac{m_{SF}}{m_C}.\frac{\rho c}{\rho_{SF}} - \cancel{(1-p).\alpha.K.}\frac{m_{SF}}{m_C}.\frac{\rho c}{\rho_{SF}} = 1$$

$$\Rightarrow \alpha \left[(1-p).k.\left(A+B\,\frac{m_{SF}}{m_C}\right) - (1-p).k - (1-p).k.\frac{m_{SF}}{m_C}\,\frac{\rho_{SF}}{\rho_C} \right]$$

$$= 1 - (1-p).k \mp (1-p).k\,\frac{m_{SF}}{m_C}.\frac{\rho_C}{\rho_{SF}}$$

$$= 1 \mp (1-p).k\left[\frac{m_{SF}}{m_C}.\frac{\rho_C}{\rho_{SF}} + 1\right]$$

$$= 1 - (1-p).\left[k\left[1 + \underbrace{\frac{m_{SF}}{m_C}.\frac{\rho_C}{\rho_{SF}}}_{=1.}\right]\right]$$

$$= 1 - 1 + p = \underline{\underline{p!}}$$

$$\Rightarrow \alpha \left[(1-p).k\left(A+B\,\frac{m_{SF}}{m_C}\right) - (1-p).k\overbrace{\left[1+\frac{m_{SF}}{m_C}\frac{\rho_C}{\rho_{SF}}\right]}^{1}\right.$$

$$= p$$

$$\Rightarrow \alpha_{eq}\left[(1-p).k\left(A+B\,\frac{m_{SF}}{m_C}\right) - (1-p)\right] = p.$$

$$\Rightarrow \alpha_{eq} = \frac{p}{(1-p).\left[k.\left[A+B\,\frac{m_{SF}}{m_C}\right] - 1\right]}.$$

$$A = \left(\frac{\rho_C}{\rho_W}.\Delta m_{GWC} + 1 + \Delta m_{CKBWC}.\frac{\rho_C}{\rho_W} - \rho_C.\Delta V_R\right.$$

$$B = \left(\frac{\rho_C}{\rho_W}.\Delta m_{GWSF} + \frac{\rho_C}{\rho_{SF}} - \rho_C.\Delta V_{RCSF}\right)$$

$$K \cdot \left(A + B \, \frac{m_{SF}}{m_C}\right) - 1$$

$$= KA + K \cdot B \cdot \frac{m_{SF}}{m_C} - 1.$$

$$= \Big(K \frac{\rho_C}{\rho_W} \cdot \Delta m_{GWC} + K + K \cdot \Delta m_{CHBWC} \frac{\rho_C}{\rho_W} - K \rho_C \Delta V_{R}$$

$$+ K \frac{\rho_C}{\rho_W} \frac{m_{SF}}{m_C} \cdot \Delta m_{GWSF} + K \frac{\rho_C}{\rho_{SF}} \frac{m_{SF}}{m_C} - K \rho_C \frac{m_{SF}}{m_C} \times \Delta V_{RCSF}$$

$$- \cancel{1} \qquad \qquad \qquad = \cancel{1}$$

$$= K \Big[\underbrace{\frac{\rho_C}{\rho_W} \cdot \Delta m_{GWC} + \Delta m_{CHBWC} \frac{\rho_C}{\rho_W} - \rho_C \Delta V_{RC}}_{A^*}$$

$$+ \frac{m_{SF}}{m_C} \Big(\underbrace{\frac{\rho_C}{\rho_W} \cdot \Delta m_{GWSF} - \rho_C \, \Delta V_{RCSF}}_{B^*} \Big) \Big]$$

$$= K \left[A^* + B^* \cdot \frac{m_{SF}}{m_C} \right]$$

$$\Rightarrow \alpha_{ajem, mwc} = \frac{p}{(1-p) \cdot K \left(A^* + B^* \frac{m_{SF}}{m_C}\right)}$$

$$A^* = 3{,}15 \times 0{,}19 + 0{,}254 \cdot 3{,}15 - 3150 \times 6{,}4 \times 10^{-5}$$

$$= 1{,}2 \; ! \quad \text{oK}$$

$$B^* = 3{,}15 \times 0{,}5 - 3150 \times 22 \times 10^{-5}$$

$$= 0{,}88 ! \quad \underline{\text{oK}}!$$